



Perfect fluid dark matter

F. Rahaman^{a,*}, K.K. Nandi^b, A. Bhadra^c, M. Kalam^d, K. Chakraborty^e

^a Department of Mathematics, Jadavpur University, Kolkata 700 032, West Bengal, India

^b Department of Mathematics, North Bengal University, Siliguri 734001, India

^c High Energy and Cosmic Ray Research Centre, University of North Bengal, Siliguri 734001, India

^d Department of Physics, Netaji Nagar College for Women, Regent Estate, Kolkata 700 032, India

^e Department of Physics, Government Training College, Hooghly 712103, West Bengal, India

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ABSTRACT

Taking the flat rotation curve as input and treating the matter content in the galactic halo region as perfect fluid we obtain a space–time metric at the galactic halo region in the framework of general relativity. We find that the resultant space–time metric is a non-relativistic dark matter induced space–time embedded in a static Friedmann–Lemaître–Robertson–Walker universe i.e. the flat rotation curve not only leads to the existence of dark matter but also suggests about the background geometry of the universe. Within its range of validity the flat rotation curve and the demand that the dark matter to be non-exotic together indicate for a (nearly) flat universe as favored by the modern cosmological observations. We obtain the expressions for energy density and pressure of dark matter there and consequently the equation of state of dark matter. Various other aspects of the solutions are also analyzed.

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1. Introduction

It has been known for a long time that rotation curves of neutral hydrogen clouds in the outer regions of galaxies cannot be explained in terms of the luminous matter content of the galaxies, at least within the context of Newtonian gravity or of general relativity (see [1] for a review). In addition, the velocity dispersion in galaxy clusters indicates a much greater mass than would be inferred from luminous matter contained in the individual galaxies. The most widely accepted explanation, based on the standard gravitational theory, postulates that almost every galaxy hosts a large amount of non-luminous matter, the so-called gravitational dark matter, consisting of unknown particles not included in the particle standard model, forming a halo around the galaxy. The dark matter provides the needed gravitational field and the mass required to match the observed galactic flat rotation curves in galaxy clusters.

The dark matter problem arises because of a naïve Newtonian analysis of the near constant tangential velocity of rotation up to distances far beyond the luminous radius of the galaxies. This leads

to the conclusion that the energy density decreases with the distance as r^{-2} and therefore that the mass of galaxies increases as $m(r) \propto r$.

There are several proposals for the dark matter component, ranging from new exotic particles such as those predicted by supersymmetry [2] to other less exotic candidates such as massive neutrinos or even ordinary celestial bodies such as Jupiter like objects. Several other analytic halo models exist in the literature including those sourced by scalar fields. Fay [3] considered modelling by Brans–Dicke massless scalar field while Matos, Guzmán and Nuñez [4] considered massless minimally coupled scalar field with a potential. A boson star formed by a self interacting massive scalar field with quartic interaction potential as a model of galactic halo was investigated by Colpi, Shapiro and Wasserman [5]. A similar boson star as a model of galactic halo was first investigated by Lee and Koh [6]. A recent halo model is discussed also in the braneworld theory [7]. Some authors [8] have considered global monopoles as a candidate for galactic halo in the framework of the scalar tensor theory of gravity.

Alternatively, explanation of the observed flat rotation curve is provided through modification of Newtonian dynamics (in the region of very small accelerations) [9,10]. Another attempt to resolve the dark matter problem lies within the framework of Weyl conformal gravity suggested by Mannheim and Kazanas [11], with the distinction that it does not postulate flat rotation curve as an input but *predict* it. Such a prediction is possible also within the Chern–Simmons gravity inspired by string theory [12,13].

* Corresponding author.

E-mail addresses: farook_rahaman@yahoo.com (F. Rahaman), kamalnandi1952@yahoo.co.in (K.K. Nandi), aru_bhadra@yahoo.com (A. Bhadra), mehedikalam@yahoo.co.in (M. Kalam), kchakraborty28@yahoo.com (K. Chakraborty).

In recent years, the amount of dark matter in the Universe has become known more precisely: CMB anisotropy data indicate that about 85% of total matter in a galaxy is dark in nature. Big bang nucleosynthesis and some other cosmological observations require that the bulk of dark matter be non-baryonic, cold or warm, stable or long-lived and not interacting with visible matter. However, despite long and intensive investigations, little is as yet known about the nature of dark matter.

The purpose of the present work is to show that, with the input of flat rotation curve and the assumption that dark matter be described as a perfect fluid, the general theory of relativity not only accounts for the dark matter component of galactic clusters but also suggests the spatial curvature of the universe. The gravitational influence of an arbitrary dark matter component is controlled by its stress tensor. In this context we note that, while the existing approaches of explaining flat rotation curves are useful in their own right, many of the models end up with predicting anisotropic dark matter fluid stress tensor. On the other hand, no physical mechanism is stated explaining why a spherical distribution should have such anisotropy. There is also no observational evidence in support of it. Therefore, it seems more reasonable to consider an isotropic perfect fluid distribution for dark matter because predictions from such model at stellar and cosmic scales have been observationally corroborated beyond any doubt.

We shall particularly study the general features of dark matter such as its equation of state. The mass distribution in galactic haloes should provide a direct probe into the nature of particles constituting the dark matter because the inner structure of the halo is particularly sensitive to the dark matter properties [14]. In this investigation, we shall work only from a fluid perspective leaving the question about the particle identity of dark matter open.

2. Gravitational field in the dark matter region

Galactic rotational velocity profiles [15] of almost all the spiral galaxies are characterized by a rapid increase from the galactic center, reaching a nearly constant velocity from the nearby region of the galaxy far out to the halo region. Our target is to exploit this observed feature to obtain the space-time metric in the halo region, and analyze it. As mentioned already, observational data suggest that the dark matter component in the galaxy accounts for almost 85% of its total mass. Naturally, luminous matter does not contribute significantly to the total energy density of the galaxy, particularly in the halo region. Therefore, we shall treat the matter in the galactic halo region as a perfect fluid defined by stresses $T_r^r = T_\theta^\theta = T_\phi^\phi = p$, where T_ν^μ is the matter energy momentum tensor.

The general static spherically symmetric space-time is represented by the following metric

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where the functions $\nu(r)$ and $\lambda(r)$ are the metric potentials. Then the Einstein field equations become ($c = 1$):

$$e^{-\lambda} \left[\frac{\lambda'}{r} - \frac{1}{r^2} \right] + \frac{1}{r^2} = 8\pi G \rho, \quad (2)$$

$$e^{-\lambda} \left[\frac{1}{r^2} + \frac{\nu'}{r} \right] - \frac{1}{r^2} = 8\pi G p, \quad (3)$$

$$\frac{1}{2} e^{-\lambda} \left[\frac{1}{2} (\nu')^2 + \nu'' - \frac{1}{2} \lambda' \nu' + \frac{1}{r} (\nu' - \lambda') \right] = 8\pi G p. \quad (4)$$

For a circular stable geodesic motion in the equatorial plane, the consideration of flat rotation curve gives the condition (see Appendix A)

$$e^\nu = B_0 r^l, \quad (5)$$

where l is given by $l = 2(\nu^\phi)^2$ and B_0 is an integration constant. The observed rotational curve profile in the region dominated by dark matter is such that the rotational velocity ν^ϕ becomes approximately a constant with $\nu^\phi \sim 10^{-3}$ (300 km/s) for a typical galaxy. Eqs. (3) and (4) then lead to the following equation

$$(e^{-\lambda})' + \frac{ae^{-\lambda}}{r} = \frac{c}{r}, \quad (6)$$

where

$$a = -\frac{4(1+l) - l^2}{2+l} \quad (7)$$

and

$$c = -\frac{4}{2+l}. \quad (8)$$

An exact solution of Eq. (6) is given by

$$e^{-\lambda} = \frac{c}{a} + \frac{D}{r^a}, \quad (9)$$

where D is an integration constant. Several observations follow from the metric (1):

(a) Note that the space-time metric given by Eq. (1) through Eqs. (5) and (9) is an interior solution. This type of space-time definitely cannot be asymptotically flat neither can it have the form of a space-time due to a centrally symmetric black hole. What can be said is that this line element describes the region where the tangential velocity of the test particles is constant and that it has to be joined with the exterior region with other types of space-time. The extent of $\nu^\phi = \text{constant}$ ends at some larger distance where the region becomes asymptotically flat. In principle, the constant D should be obtained from the junction conditions but the galactic boundary is not observationally defined yet. It is more likely that dark matter distribution will not remain the same near the periphery of the halo.

(b) Inserting the above solution in Eqs. (2)–(4), one can readily get the expressions for ρ , p as

$$\rho = \frac{1}{8\pi G} \left[\frac{l(4-l)}{4+4l-l^2} r^{-2} - \frac{D(6-l)(1+l)}{2+l} r^{l(2-l)/(2+l)} \right], \quad (10)$$

$$p = \frac{1}{8\pi G} \left[\frac{l^2}{4+4l-l^2} r^{-2} + D(1+l) r^{l(2-l)/(2+l)} \right]. \quad (11)$$

The most notable aspect is the presence of the last term in the expressions of both energy density ρ and pressure p that increases with radial distance (from galactic center). Note that according to the Newtonian theory, which is supposed to be indistinguishable from general relativity in very weak field, one should expect only the term $\rho_{\text{Newton}} = \frac{1}{8\pi G} \frac{l}{r^2}$ in agreement with the Poisson equation, which integrates to give the Newtonian mass $M(r)$ increasing linearly with r . But the input of the constant tangential velocity leads to general relativistic corrections to Newtonian expressions as evident in Eqs. (10), (11). The first term of the right-hand side of Eq. (10) gives the expected Newtonian term in the leading order and to the same order a general relativistic correction term $\frac{1}{8\pi G} \frac{5l^2}{4r^2}$. Since l is small, the correction terms are small as expected.

However, the second term in Eq. (10) has completely different nature than the conventional feature of dark matter energy density; the corresponding mass increases *nonlinearly* with radial distance r , though a bit slowly as l is very small. This contribution will vanish if D vanishes. We would see that even $D = 0$ is consistent with the flat rotation curve.

(c) The parameter D is recognized¹ as the spatial curvature (with a negative sign) of the universe. A comparison of the obtained space-time metric in the limit $l \rightarrow 0$ with the static Friedmann–Lemaître–Robertson–Walker (FLRW) metric [$ds^2 = -dt^2 + \frac{1}{1-kr^2} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$] leads to the identification that $D = -k$. Since our input was only the flat rotation curve and we have not considered anything regarding cosmological spatial curvature while deriving the metric, the appearance of D in the metric element is quite interesting. The solution thus may be thought of dark matter induced space-time embedded in a static FLRW metric. In general $l \neq 0$ and hence a is not equal to -2 , and the interpretation of spatial curvature does not seem at all evident.

An immediate question is how a spatially curved FLRW universe could be static. If it is due to balance of the curvature term by some other fluid with equation of state $\rho = -3p$, the question of stability will arise. It seems that we have ended up with a static universe because right from the beginning we looked for a static solution (all the metric element are considered time independent). While working on a local problem (flat rotation curve), the universe is usually considered at any particular epoch fixing the scale factor $R(t)$ to be constant (often normalized to unity, $R(t_0) = 1$ at the present epoch). The dynamicity of the FLRW metric is thus absent in local gravitational phenomena. However, we see in the present work that the curvature effect has appeared even in local gravitational phenomena. It would certainly be an interesting but challenging task to study the stability criterion in this configuration see footnote 1. We reserve it for a separate communication.

(d) The equation of state parameter ω for the effective fluid (dark matter plus “curvature fluid”), can be obtained directly from Eqs. (10) and (11), which is given by

$$\frac{p}{\rho} = \omega = \frac{l^2 r^a + D(1+l)(4+4l-l^2)}{l(4-l)r^a - [D(6-l)(1+l)(4+4l-l^2)/(2+l)]}. \quad (12)$$

Since l is small, effectively $\omega \simeq \frac{l^2 r^a + 4D}{4l r^a - 12D}$. If the total matter in the flat rotation curve region of galaxies has to be non-exotic, i.e. if the dark matter satisfies the known energy conditions, ω for the total matter content must be positive. This is achieved only if we set $l^2 r^a/4 < D < l r^a/3$. Note that our discussion is restricted only in the region of flat rotation curve of galaxies for which r is typically between few ten kpc to few hundred kpc and as mentioned already v^ϕ is few hundred km/s. Hence positive ω implies that D is nearly zero, if not exactly. Fig. 1 shows the variation of ω with D for a typical distance $r = 200$ kpc and $v^\phi = 300$ km/s.

As may be seen from Fig. 1, the limit on D is much stringent compared to the (cosmological) observational restriction. The equation of state of the dark matter component may be obtained by taking $D = 0$ in the above equation which gives

$$\omega = \frac{l}{(4-l)} \simeq 2.5 \times 10^{-7}. \quad (13)$$

Thus $p \ll \rho$ for dark matter implying its non-relativistic nature, which is a well-known fact. Further ω is positive, which means that the fluid is non-exotic. On the other hand, the equation of state of ‘curvature fluid’ is found to be $\rho + 3p = 0$ from Eq. (12) in the limit $l \rightarrow 0$, which is a familiar result.

(e) Note that we can rewrite e^λ in the standard Schwarzschild form

$$e^\lambda = \left[1 - \frac{2m(r)}{r} \right]^{-1} \quad (14)$$

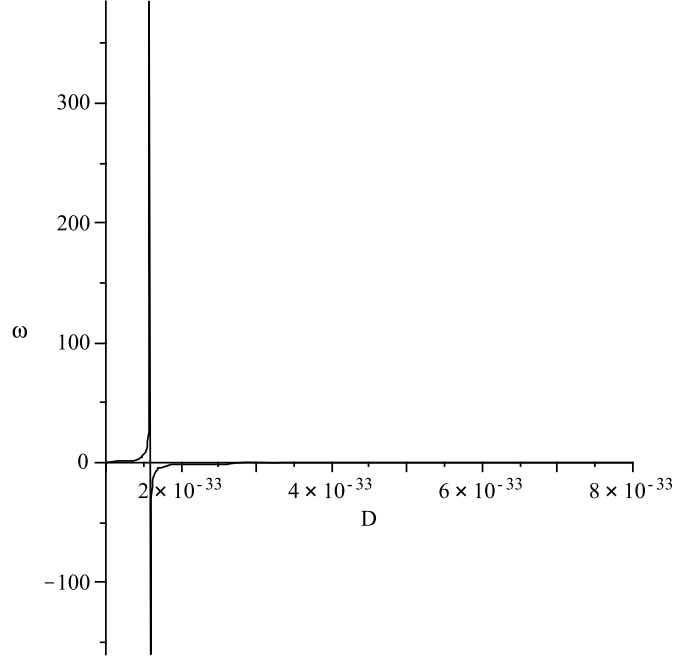


Fig. 1. The variation of ω with D for a typical distance $r = 200$ in kpc.

which is often convenient. Such a form has the advantage that it immediately reveals not only the mass parameter $m(r)$ but also shows that the proper radial length is larger than the Euclidean length because $r > 2m(r)$. But most importantly, this inequality is essential for signature protection, which dictates that $e^\lambda > 1$. This is a crucial condition to be satisfied by any valid metric. Now, from the metric function (9), for $D = 0$, we get

$$e^\lambda = 1 + \frac{4l - l^2}{4} > 1. \quad (15)$$

This implies that the essential requirement is fulfilled for this value of D .

(f) Let us rewrite the metric (1) for $D = 0$ under the radial rescaling

$$r = \sqrt{\frac{c}{a}} r', \quad (16)$$

which yields

$$ds^2 = -B'_0 r'^l dt^2 + dr'^2 + \left(\frac{c}{a}\right) r'^2 (d\theta^2 + \sin^2\theta d\phi^2),$$

$$B'_0 = B_0 \left(\sqrt{\frac{c}{a}} \right)^l \quad (17)$$

showing a surplus angle in the surface area given by

$$S_1 = 4\pi r^2 \cdot \frac{c}{a} = \frac{4\pi r^2}{1 + 2(v^\phi)^2 - (v^\phi)^4}. \quad (18)$$

If the probe particles were photons, so that $v^\phi = 1$, the surface area would remain finite but reduced to half of the spherical surface area $S_2 = 4\pi r^2$. This is an interesting result, which distinguishes itself from that in the massless scalar field model where $S \rightarrow \infty$ as $v^\phi = 1$ [4]. For a typical rotational velocity of $v^\phi = 10^{-3}$ in the galactic halo region, the difference of the two surface areas

$$S_2 - S_1 = 4\pi r^2 \left[\frac{2(v^\phi)^2 - (v^\phi)^4}{1 + 2(v^\phi)^2 - (v^\phi)^4} \right] \quad (19)$$

¹ We sincerely thank the referee for raising this interesting possibility.

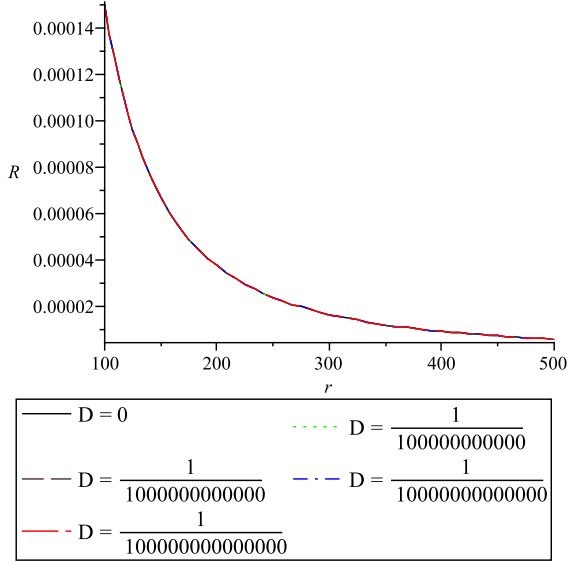


Fig. 2. The variation of R with r in kpc for small different values of D . We choose $v^\phi \sim 10^{-3}$ (300 km/s) for a typical galaxy.

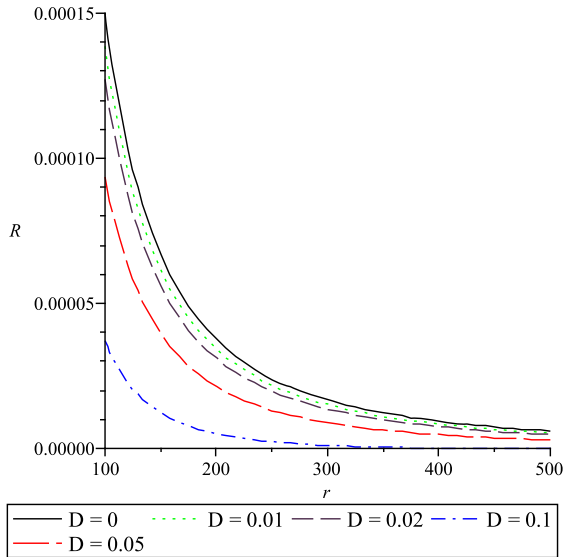


Fig. 3. The variation of R with r in kpc for large different values of D . We choose $v^\phi \sim 10^{-3}$ (300 km/s) for a typical galaxy.

grows as $\sim 10^{-6}$ in units of flat surface area, which indicates another deviation from the massless scalar model in which it grows as $\sim 10^{-12}$ [4].

(g) The Ricci scalar for the derived space-time is given by

$$R = \frac{Da^2(4+l) - (aD + cr^a)(l^2 + 2l + 4) + 4ar^a}{2ar^{2+a}}. \quad (20)$$

As $l \rightarrow 0$, $R = -6D$, once again suggesting that D is the spatial curvature. We plot R vs r . One may note that the value of R is small. For small different values of D , one cannot distinguish the variations of R with respect to r (see Fig. 2). But for higher values of D , Fig. 3 shows a sharp variation of R . Note that large values of D imply $\omega(r) < 0$, i.e., dark matter has to be exotic in nature.

3. Other aspects

3.1. The total gravitational energy

One notes from Eq. (12) that the halo matter is not exotic in nature and consequently, we expect attractive gravity in the halo. Following the suggestion given by Lyndell-Bell et al. [17], we calculate the total gravitational energy E_G between two fixed radii, say, r_1 and r_2 :

$$E_G = M - E_M = 4\pi \int_{r_1}^{r_2} [1 - \sqrt{e^{\lambda(r)}}] \rho r^2 dr \quad (21)$$

$$= 4\pi \int_{r_1}^{r_2} \left[1 - \sqrt{\frac{1}{\frac{c}{a} + \frac{D}{r^a}}} \right] \left[\frac{1}{8\pi G} \left(\frac{D(a-1)}{r^{a+2}} + \frac{(1-\frac{c}{a})}{r^2} \right) \right] r^2 dr, \quad (22)$$

where

$$M = 4\pi \int_{r_1}^{r_2} \rho r^2 dr \quad (23)$$

is the Newtonian mass given by

$$M = 4\pi \int_{r_1}^{r_2} \rho r^2 dr = \frac{1}{G} \left[\frac{(1-\frac{c}{a})r}{2} - \frac{D}{2r^{a-1}} \right]_{r_1}^{r_2}. \quad (24)$$

Thus we get the total gravitational energy as

$$E_G = \frac{1}{G} \left[\frac{(1-\frac{c}{a})r}{2} - \frac{D}{2r^{a-1}} - \left(1 - \frac{c}{a} \right) \frac{r F[(0.5, \frac{1}{a}); (1 + \frac{1}{a}); -\frac{ar^a D}{c}] }{\sqrt{\frac{c}{a}}} \right]_{r_1}^{r_2} + \frac{1}{G} \left[D(1-a)r^{(-0.5a+1)} \times \frac{F[(-0.5 + \frac{1}{a}, 0.5); (0.5 + \frac{1}{a}); -\frac{cr^a}{aD}] }{\sqrt{Da}(-0.5 + \frac{1}{a})} \right]_{r_1}^{r_2}. \quad (25)$$

Figs. 4 and 5 show that the total gravitational energy is small but negative whether we choose D non-zero or zero for arbitrary $r_2 > r_1 > 0$. Thus for the existence of non-exotic matter in the halo, we recommend the value of $D \leq 10^{-11}$. In the distant halo region we have taken, typically, $r \sim 200$ kpc in the figures below.

3.2. Attraction

Now we study the geodesic equation given by

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma_\alpha^{\mu\gamma} \frac{dx^\mu}{d\tau} \frac{dx^\gamma}{d\tau} = 0 \quad (26)$$

for a test particle that has been ‘placed’ at some radius r_0 . This yields the radial equation

$$\frac{d^2 r}{d\tau^2} = -\frac{1}{2} \left[\frac{c}{a} + \frac{D}{r^a} \right] \times \left[\frac{Da}{r^{a+1}} \left(\frac{c}{a} + \frac{D}{r^a} \right)^{-2} \left(\frac{dr}{d\tau} \right)^2 + B_0 l r^{l-1} \left(\frac{dt}{d\tau} \right)^2 \right]_{r=r_0}, \quad (27)$$

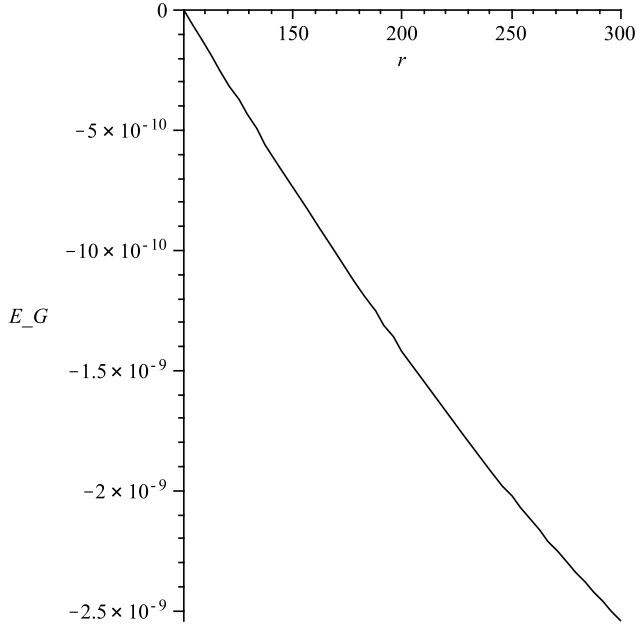


Fig. 4. The variation of E_G with r in kpc. The lower limit of integration in Eq. (24) fixed at, say, $r_1 = 100$ kpc while r_2 is varied from 100 to 500 kpc. We choose $G = 1$, $D = 10^{-11}$ and $v^\phi \sim 10^{-3}$ (300 km/s) for a typical galaxy.

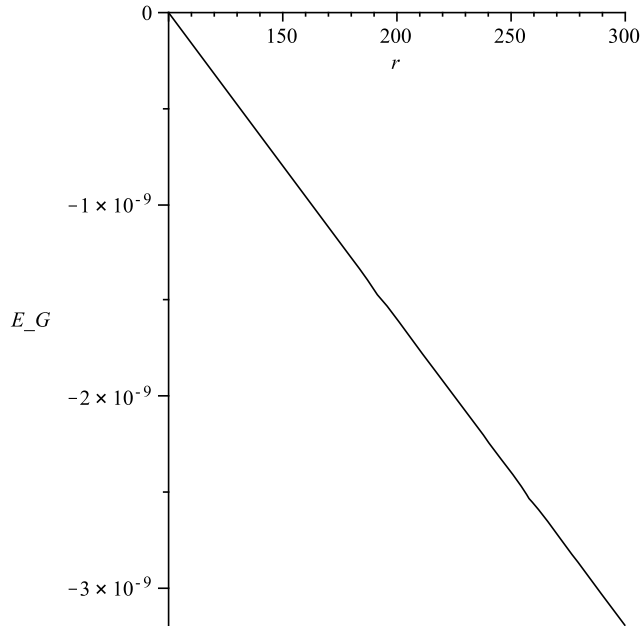


Fig. 5. The variation of E_G with r in kpc. The lower limit of integration in Eq. (24) fixed at, say, $r_1 = 100$ kpc while r_2 is varied from 100 to 500 kpc. We choose $G = 1$, $D = 0$ and $v^\phi \sim 10^{-3}$ (300 km/s) for a typical galaxy.

which is negative as the quantity in the square bracket is positive. Thus particles are attracted towards the center. This result is in agreement with the observations i.e., gravity on the galactic scale is attractive (clustering, structure formation etc.).

3.3. Stability

Let us define the four velocity $U^\alpha = \frac{dx^\alpha}{d\tau}$ for a test particle moving solely in the space of the halo (restricting ourselves to $\theta = \pi/2$), the equation $g_{\nu\sigma} U^\nu U^\sigma = -m_0^2$ can be cast in a Newtonian form

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 + V(r) \quad (28)$$

which gives

$$V(r) = -\left[E^2 \left\{1 - \frac{r^{-l} \left[\frac{c}{a} + \frac{D}{r^a}\right]}{B_0}\right\} + \left[\frac{c}{a} + \frac{D}{r^a}\right] \left(1 + \frac{L^2}{r^2}\right)\right], \quad (29)$$

$$E = \frac{U_0}{m_0}, L = \frac{U_3}{m_0}, \quad (30)$$

where the constants E and L , respectively, are the conserved relativistic energy and angular momentum per unit rest mass of the test particle. Circular orbits are defined by $r = R = \text{constant}$ so that $\frac{dR}{d\tau} = 0$ and, additionally, $\frac{dV}{dr}|_{r=R} = 0$. From these two conditions follow the conserved parameters:

$$L = \pm \sqrt{\frac{l}{2-l}} R \quad (31)$$

and using it in $V(R) = -E^2$, we get

$$E = \pm \sqrt{\frac{2B_0}{2-l}} R^{\frac{l}{2}}. \quad (32)$$

The orbits will be stable if $\frac{d^2V}{dr^2}|_{r=R} < 0$ and unstable if $\frac{d^2V}{dr^2}|_{r=R} > 0$. Putting the expressions for L and E in $\frac{d^2V}{dr^2}|_{r=R}$, we obtain, after straightforward calculations, the final result, viz.,

$$\frac{d^2V}{dr^2}\bigg|_{r=R} = -\left[\frac{2cl(1-l)}{a(2-l)R^2} + \frac{D(16l+8l^2-4l^3+2l^4)}{(2-l)(2+l)^2} R^{\frac{l(2-l)}{2+l}}\right]. \quad (33)$$

One may note that, for $D = 0$ as well as for $D \neq 0$, $\frac{d^2V}{dr^2}|_{r=R} < 0$. So the circular orbits are always stable.

4. Conclusions

It is well known that observation of flat rotation curve suggests that a substantial amount of non-luminous dark matter is hidden in the galactic halo. Here we have found that the flat rotation curve suggests also the background geometry of the universe. The space-time geometry we have obtained can be interpreted as the one due to dark matter embedded in the static FLRW universe. This is probably the first indication that the spatial curvature of the universe can be obtained from a local gravitational phenomenon.

If we demand that matter in the flat rotation curve region be non-exotic (i.e., obey the usual energy conditions), we obtain the result that the universe should be nearly flat, if not exactly so, which is consistent with modern cosmological observations. However, one should note that the flat rotation curves have been observed only up to the outskirts of galactic halos, not out to cosmological scales. Hence the notion of flat universe as favored by the present analysis should be taken more as suggestive than conclusive.

The equation of state of the dark matter component has been obtained by treating it as perfect fluid and the expressions for general relativistic correction terms for the pressure and energy density over those obtained from the Newtonian theory are also derived. The corrections are, however, small as expected.

The geodesic equation [Eq. (26)] of a test particle in the derived space-time suggests that the particle will be attracted towards the center. We have quantified the attractive effect in the relativistic case by calculating the total gravitational energy E_G (which is negative) in the halo region. We have also demonstrated the stability of the circular orbits in our space-time solution. Thus our solution

satisfies two crucial physical requirements – stability of circular orbits and attractive gravity in the halo region. Investigation on other observational constraints on the model is underway.

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Appendix A

To derive tangential velocity of circular orbits, we start with the line element

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

The Lagrangian for a test particle reads

$$2L = -e^{\nu(r)} \dot{t}^2 + e^{\lambda(r)} \dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2).$$

We guess the conserved quantities, the energy $E = e^{\nu(r)} \dot{t}$, the ϕ -momentum $L_\phi = r^2 \sin^2\theta \dot{\phi}$, and the total angular momentum, $L^2 = L_\theta^2 + (\frac{L_\phi}{\sin\theta})^2$, with $L_\theta = r^2 \dot{\theta}$. The radial motion equation can be written as:

$$\dot{r}^2 + V(r) = 0$$

with the potential $V(r)$ given by

$$V(r) = -e^{-\nu(r)} \left(e^{-\lambda(r)} E^2 - \frac{L^2}{r^2} - 1 \right).$$

For circular orbits, we have the conditions, $\dot{r} = 0$, $V_r = 0$ and $V_{rr} > 0$ [16]. These imply the following expressions for the energy and total momentum of the particles in circular orbits:

$$E^2 = \frac{2e^{2\nu(r)}}{2e^{\nu(r)} - r(e^{\nu(r)})_r},$$

$$L^2 = \frac{r^3(e^{\nu(r)})_r}{2e^{\nu(r)} - r(e^{\nu(r)})_r}.$$

The second derivative of the potential evaluated at the extrema

$$V(r)_{rr}|_{\text{extrema}} = 2 \frac{\frac{r(e^{\nu(r)})_{rr}}{e^{\nu(r)}} + \frac{(e^{\nu(r)})_r}{e^{\nu(r)}} \left(3 - \frac{r(e^{\nu(r)})_r}{e^{\nu(r)}} \right)}{re^{\lambda(r)} \left(2 - \frac{r(e^{\nu(r)})_r}{e^{\nu(r)}} \right)}.$$

Now, the tangential velocity

$$(v^\phi)^2 = r^2 e^{-\nu(r)} (\dot{\theta}^2 + \sin^2\theta \dot{\phi}^2)$$

can be obtained for particles in stable circular orbits as

$$(v^\phi)^2 = \frac{r(e^{\nu(r)})_r}{2e^{\nu(r)}}.$$

Using the tangential velocity to be constant for several radii, the above expression yields

$$e^\nu = B_0 r^l$$

where l is given by $l = 2(v^\phi)^2$ and B_0 is an integration constant.

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